

6/MTH-352 Syllabus-2023

2026

(May-June)

FYUP : 6th Semester Examination

MATHEMATICS

(Discrete Mathematics)

(MTH-352)

Marks : 75

Time : 3 hours

*The figures in the margin indicate full marks
for the questions.*

Answer **four** questions, choosing **one** from each Unit.

UNIT—I

1. (a) A palindrome is a word that reads the same forward and backward. How many seven-letter palindromes can be made out of the English alphabet?

3

- (b) Use pigeonhole principle to prove that if 9 balls are to be kept in 4 containers, there must be at least one container which contains at least 3 balls.

4

- (c) Use the principle of mathematical induction to prove that

$$1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$$

5

- (d) Solve the recurrence relation

$$a_{n+2} - 5a_{n+1} + 6a_n = 2$$

with initial conditions $a_0 = 1$ and $a_1 = -1$.

6

2. (a) Find the total number of distinct permutations of the word 'MATHEMATICS'.

3

- (b) Use the principle of mathematical induction to prove that

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

4

- (c) Using the principle of inclusion and exclusion, find the number of integers between 1 and 200 that are divisible by 3 or 4 or 5.

5

- (d) Let $p(n, k)$ denote the number of partitions of n into k parts. Prove that $p(n, k) = p(n-1, k-1) + p(n-k, k)$. Hence deduce that $p(6, 3) = 3$. 4+2=6

UNIT—II

3. (a) Draw the Hasse diagram of the relation R , where

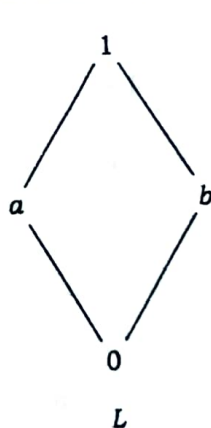
$$A = \{1, 2, 3, 4\}$$

$$R = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 4), (3, 3), (3, 4), (4, 4)\}$$

3

- (b) Prove that the direct product of two lattices is a lattice. 8

- (c) Give the definitions of lattice and lattice homomorphism. Determine whether there exists a lattice homomorphism from the lattices L and M represented by the following Hasse diagrams : 1+2+5=8

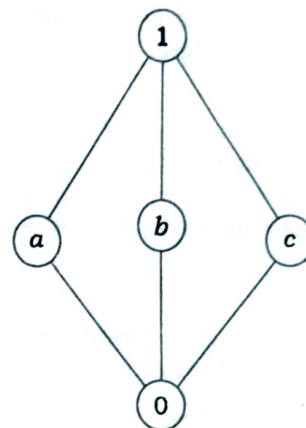


4. (a) Show that the union of two sublattices is not necessarily a sublattice. 3
- (b) State the principle of duality in lattices and write the dual of $a \vee (a \wedge b)$. 4
- (c) Let $A = \{1, 2, 3, 4, 6, 12\}$ be partially ordered by the divisibility relation $|$.
- (i) Draw the Hasse diagram of the poset $(A, |)$.
- (ii) Show that $(A, |)$ is a lattice. 3+3=6
- (d) Let S be a set. Prove that the poset $(\mathcal{P}(S), \subseteq)$ is a complete lattice, where $\mathcal{P}(S)$ stands for the power set of S . 6

UNIT—III

5. (a) Define a Boolean algebra. Provide an example. 2+2=4
- (b) Minimize the Boolean function
- $$f = x'y'z + x'yz' + x'yz + xy'z + xyz$$
- using Quine-McCluskey method. 7

- (c) Prove that the lattice shown in the figure given below is modular but not distributive : 5+3=8



6. (a) Express the Boolean expression $x(y(z+w) + z(u+v))$ as a switching circuit. 3
- (b) Construct the truth table for the Boolean expression $xy' + z(x' + y)$. 4
- (c) Prove that $(a + b)' = a'b'$, where a, b are elements of a Boolean algebra. 4
- (d) Minimize $g(a, b, c) = a'b'c' + ab'c' + abc'$ using Karnaugh diagram. 8

UNIT—IV

7. (a) Prove that the sum of the degrees of all vertices of a graph equals twice the number of edges. 5
- (b) Draw a simple graph with 5 vertices having degrees 3, 3, 3, 3, 4. Does there exist a simple graph with five vertices having degrees 2, 2, 4, 4, 4? Justify. $2+3=5$
- (c) Define complete graph. Show that a complete graph with n vertices consists of $\frac{n(n-1)}{2}$ edges. $1+4=5$
- (d) Define isomorphism of graphs with an example. Is the complete bipartite graph $K_{2,3}$ isomorphic to the complete graph K_5 ? Justify. $2+2=4$
8. (a) Prove that if a connected graph G is Eulerian, then every vertex of G has even degree. 5
- (b) Prove that there is a unique path between every pair of vertices in a tree. 4

- (c) Let G be a connected graph with at least two vertices. If the number of edges in G is less than the number of vertices, then prove that G has a vertex of degree one. 5
- (d) Define adjacency matrix and incidence matrix of a graph G . Determine the adjacency matrix of the complete bipartite graph on the sets $U = \{v_1, v_2\}$ and $V = \{v_3, v_4, v_5\}$. 5
